Functional Encryption for Inner Product Predicates from Learning with Errors

Shweta Agrawal[1], **David Mandell Freeman**[2], and Vinod Vaikuntanathan[3]

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Seoul, Korea
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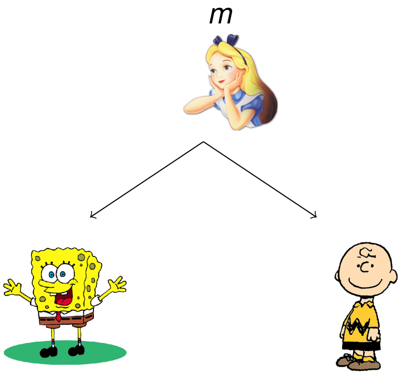
Traditional Public-Key Encryption [DH76,RSA78,...]



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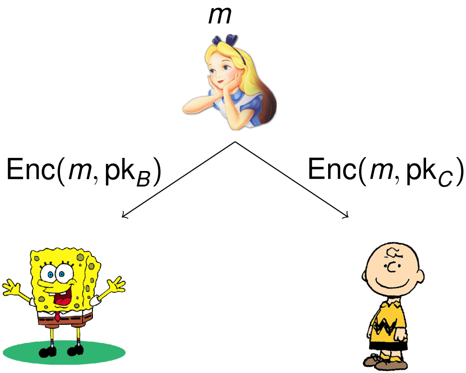

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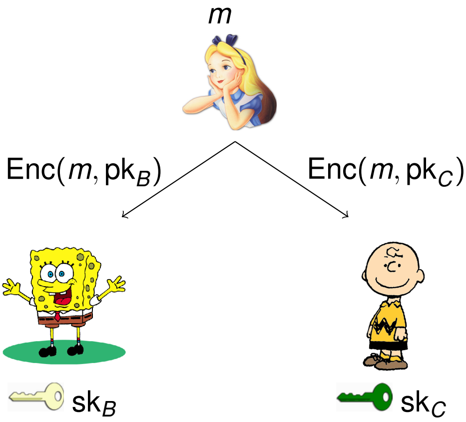

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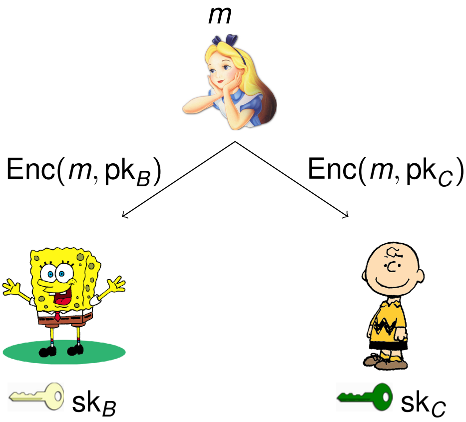
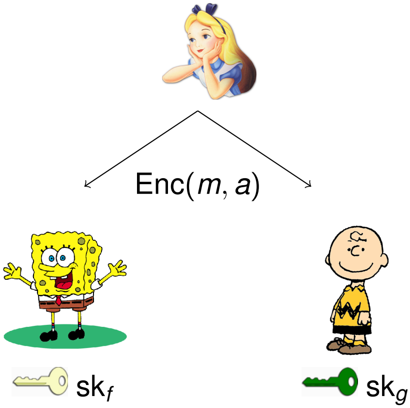


- m must be encrypted separately to each user.
- Recipient set must be decided in advance.

Functional Encryption [SW05,GPSW05,BW07,KSW08,...]



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- sk equipped with predicate f.
- User with sk_f can decrypt iff $f(a) = 1$.

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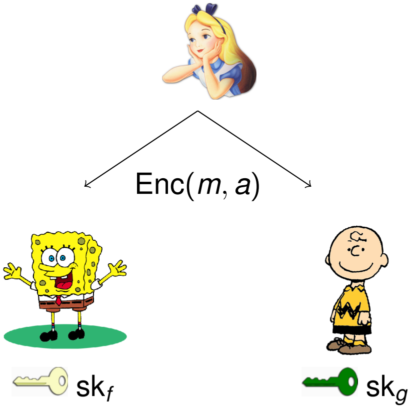

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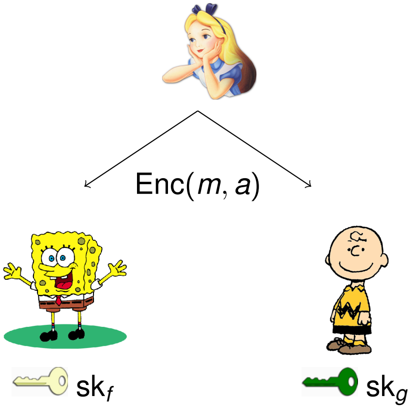

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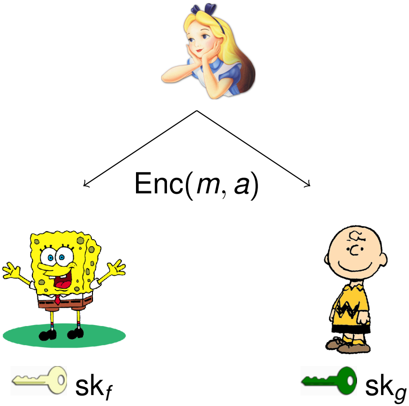

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$g =$ (conf="Eurocrypt" OR year=2011)

Prior Work on Functional Encryption

Identity-based encryption is functional encryption for equality predicates.

- Ciphertexts & keys equipped with identity *id*.
- Decrypt succeeds iff (key *id*) $=$ (CT *id*).
- Achieved using pairings, QR, and lattices.
  [BF01,BB04ab,...], [C01,BGH07], [GPV08,CHKP10,ABB10ab]

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Inner product predicates [KSW08,OT09,LOSTW10,...]:

- CT $\leftrightarrow$ vector $\vec{w}$; key $\leftrightarrow$ vector $\vec{v}$
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[KSW08]: Inner product predicates allow us to instantiate range, conjunction, disjunction, and polynomial evaluation predicates.

Our Contribution

Functional encryption for inner product predicates based on the *learning with errors* (LWE) assumption.

- Achieves functionality of [KSW08].
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- Allows inner products over small fields.

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Privacy property: CT attribute is hidden from users who cannot decrypt ("*weakly attribute hiding*").

- [KSW08] construction hides attribute from all users.
- Open problem: achieve same privacy property from LWE.

Building Blocks

Lattice-based PKE [GPV08 “dual Regev”]:

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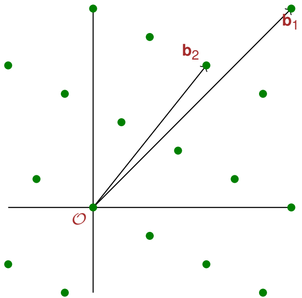

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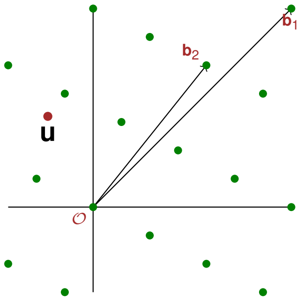

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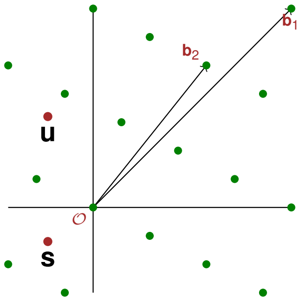

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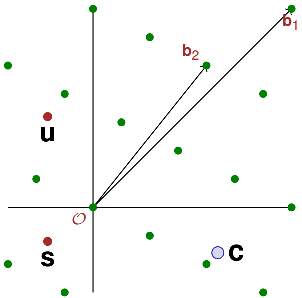

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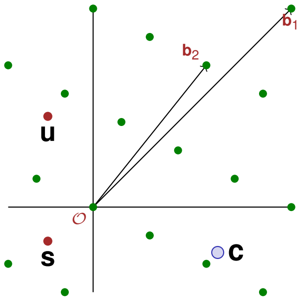

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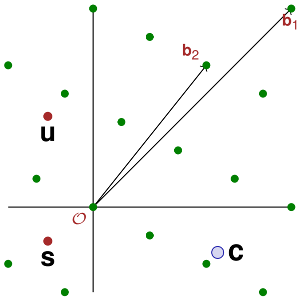

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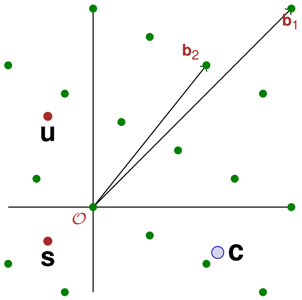

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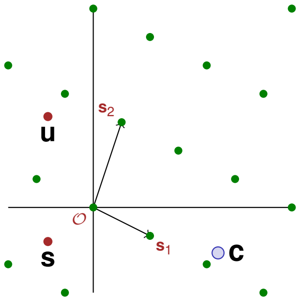

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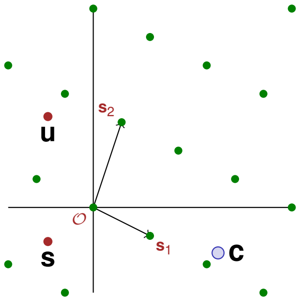

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[A99,AP09]: Can generate a random lattice Λ along with short basis of $\Lambda^\perp$ = trapdoor for Λ.

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Each identity *id* defines a lattice Λ_{id}.

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  CT for $\vec{w}$ decryptable by sk for any $\vec{v}$ with $\langle \vec{v}, \vec{w} \rangle = 0$.

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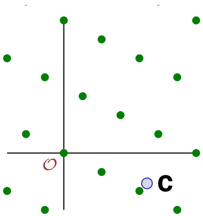
Conclude: can't require CT lattice to match sk lattice.

Building Functional Encryption

Encrypt relative to
attribute lattice $\Lambda_{\vec{w}} \subset \mathbb{Z}^r$



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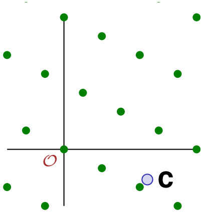

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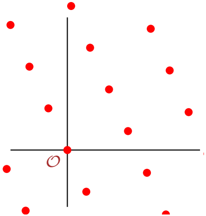

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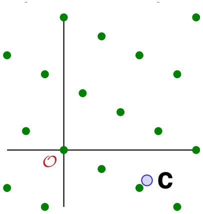

"short"
linear map

$T_{\vec{v}} : \mathbb{Z}^r \to \mathbb{Z}^s$

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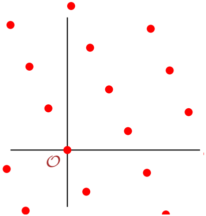

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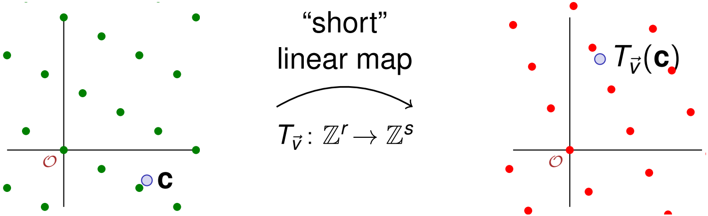

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Encrypt relative to *attribute lattice* $\Lambda_{\vec{w}} \subset \mathbb{Z}^r$

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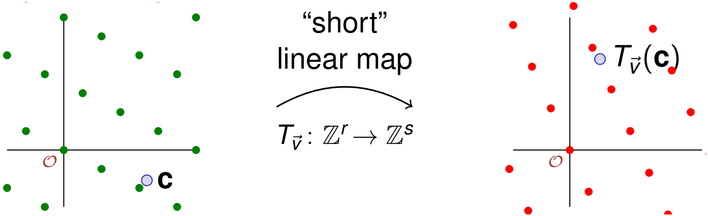

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$\Rightarrow$ key for $\Lambda_{\vec{v}}$ can decrypt $T_{\vec{v}}(\mathbf{c})$.

What Lattices are Used?

Regev/GPV lattice Λ defined by matrix $\mathbf{A}_0 \in \mathbb{Z}_q^{n \times m}$, $n < m$:

$$\Lambda = \Lambda_q(\mathbf{A}_0) = \{\mathbf{v} \in \mathbb{Z}^m : \mathbf{v} \bmod q = \mathbf{r}^t \cdot \mathbf{A}_0 \text{ for some } \mathbf{r} \in \mathbb{Z}_q^n\}$$

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[ABB10a] IBE: to encrypt to identity *id*, use lattice

$$\Lambda_{id} = \Lambda_q(\mathbf{A}_0 \parallel \mathbf{A}_1 + H(id)\mathbf{B}) \subset \mathbb{Z}^{2m}.$$

- public $\mathbf{A}_0, \mathbf{A}_1, \mathbf{B} \in \mathbb{Z}_q^{n\times m}$.
- $H\colon \{0,1\}^* \to \mathbb{Z}_q^{n\times n}$ is a hash function.

Secret key for Λ_{id} can be computed using trapdoor for $\mathbf{A}_0$.

A Functional Encryption Scheme

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To decrypt, apply transformation $T_{\vec{v}} : \mathbb{Z}^{(1+\ell)m} \to \mathbb{Z}^{2m}$ given by

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Then

$$T_{\vec{v}}(\Lambda_{\vec{w}}) = \Lambda_q(\mathbf{A}_0 \parallel \sum v_i\mathbf{A}_i + \langle \vec{v}, \vec{w} \rangle \mathbf{B})$$

So sk for $\Lambda_{\vec{v}}$ can decrypt $T_{\vec{v}}(CT)$ iff $\langle \vec{v}, \vec{w} \rangle = 0$ (and $\vec{v}$ is short).

(Selective) Security Model

Challenger

Adversary

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attributes $\vec{w}_0, \vec{w}_1$ $\longleftarrow$

Challenger

Adversary

(Selective) Security Model

Challenger

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attributes $\vec{w}_0, \vec{w}_1$ ←

 pk →

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 pk →

predicate $\vec{v} : \langle \vec{v}, \vec{w}_b \rangle \neq 0$ ←

Challenger

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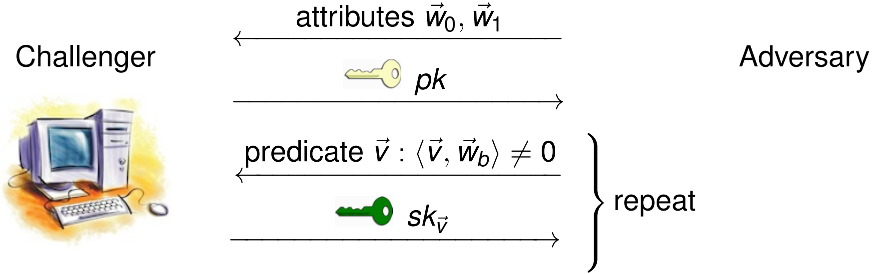

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} repeat

messages m_0, m_1 ←

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$\mathrm{Enc}(m_b, \vec{w}_b)$ →

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b $\xleftarrow{R} \{0,1\}$

Adversary

attributes $\vec{w}_0, \vec{w}_1$ ←

 pk →

predicate $\vec{v} : \langle \vec{v}, \vec{w}_b \rangle \neq 0$ ←

 $sk_{\vec{v}}$ →

} repeat

messages m_0, m_1 ←

$\text{Enc}(m_b, \vec{w}_b)$ →

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Definition

Scheme is *weakly attribute hiding* if $\left| \Pr[b' = b] - \frac{1}{2} \right|$ is negligible for all efficient $\mathcal{A}$.

Security Theorem

Learning With Errors (LWE) assumption [R05]

For fixed $\mathbf{s} \in \mathbb{Z}_q^n$, "noisy inner products" with $\mathbf{s}$ are indistinguishable from random:

$$\{\mathbf{a}_i, \langle \mathbf{s}, \mathbf{a}_i \rangle + e_i\}_{i=1}^{m} \approx_c \{\mathbf{a}_i, r_i\}_{i=1}^{m}$$

for random $\mathbf{a}_i \in \mathbb{Z}_q^n$, small $e_i \in \mathbb{Z}$, and random $r_i \in \mathbb{Z}_q$.

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Theorem

If the LWE assumption holds, then our inner product encryption scheme is weakly attribute hiding.

Proof Idea

CT lattice: $\Lambda_{\vec{w}} = \Lambda_q(\mathbf{A}_0 \parallel \mathbf{A}_1 + w_1\mathbf{B} \parallel \cdots \parallel \mathbf{A}_\ell + w_\ell\mathbf{B})$.
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Embed LWE challenge in the matrix $\mathbf{A}_0$.

- If LWE challenge is "noisy inner products" $\langle \mathbf{s}, \mathbf{a}_i \rangle + e_i$, obtain real CT.
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Adversary that breaks system can break LWE assumption.

Open Questions

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Thank you!